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LOW ENERGY YUKAWA INPUT PARAMETERS FOR YUKAWA COUPLING UNIFICATION

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I make a short review of the most recent determinations of the quark masses and run them to the M_Z energy scale

GUT and SUSY theories predict some relations among the fermion masses (or more properly among Yukawa couplings) at the unification scale, $M_{GUT} \sim 10^{16}(GeV)$. For instance, in $SU(5)$ we have the usual lepton-bottom quarks unification, $h_b = h_\tau$, $h_s = h_\mu$, $h_d = h_e$, or the modified Georgi-Jarlskog relation, $h_b = h_\tau$, $h_s = h_\mu/3$, $h_d = 3h_e$, while for $SO(10)$ typically we get unification of the third family, $h_t = h_b = h_\tau = h_{\nu_\tau}$. This together with the RGE's provide us a powerful tool for predicting quark masses at low energies.

In the following I will not talk about Unification but rather try to make a short review of the most recent determinations of the quark masses and to do the running to M_Z , the mass of the Z boson. Why M_Z ?, because for model building purposes it is a good idea to have a reference scale, because extensions of the SM appear above M_Z and because below M_Z the strong coupling constant α_s is really strong and then special care has to be taken on the running and the matching in passing a heavy quark threshold.

Because of confinement to define what is the mass of a quark is not an easy task. For leptons it is clear that the physical mass is the pole of the propagator but for quarks we need a precise theoretical framework and the masses appear more like coupling constants. Dealing with quark masses we can find the Euclidean mass, $M_E(p^2 = -M^2)$, defined as the mass renormalized at the Euclidean point $p^2 = -M^2$, it is gauge dependent but softly dependent on Λ_{QCD} . However, it does not appear in the most recent works. We can talk about the “perturbative” pole mass, $M(p^2 = M^2)$, perturbative because only order by order in perturbation theory the pole of the propagator is well defined. It is gauge and scheme independent but appears to be ambiguous because of non-perturbative renormalons. Finally $\bar{m}(\mu)$, the mass renormalized in the

Table 1: Recent determinations of the light quark masses from second order χ PT, QCD Sum Rules and lattice.

J. Gasser H. Leutwyler	χ PT	$O(p^4)$	$\frac{m_d - m_u}{m_s - \hat{m}} \frac{2\hat{m}}{m_s + \hat{m}} = 2.35 \times 10^{-3}$ $m_s/\hat{m} = 25.7 \pm 2.6$	
Donoghue et al.	χ PT	$O(p^4)$	$\frac{m_d - m_u}{m_s - \hat{m}} \frac{2\hat{m}}{m_s + \hat{m}} = 2.11 \times 10^{-3}$ $m_s/\hat{m} = 31.$	
Bijnens Prades de Rafael	FESR Laplace SR (pseudo)	NNLO	$(\bar{m}_u + \bar{m}_d)(1GeV)$ $= 12.0 \pm 2.5$	$\alpha_s(M_Z) = 0.117(5)$
Ioffe et al.	Isospin viol. in QCD SR	NNLO	$(\bar{m}_d - \bar{m}_u)(1GeV)$ $= 3.0 \pm 1.0$	$\Lambda = 150.$
S. Narison	τ -like SR	NNLO NLO	$\bar{m}_s(1GeV) = 197(29)$ $\bar{m}_s(1GeV) = 222(22)$	$\alpha_s(M_Z) = 0.118(6)$
M. Jamin M. Münz	QSSR (scalar)	NNLO	$\bar{m}_s(1GeV) = 189(32)$	$\alpha_s(M_Z) = 0.118(6)$
Chetyrkin et al.	QCD SR	NNLO	$\bar{m}_s(1GeV) = 171(15)$	$\alpha_s(M_Z) = 0.117(5)$
Allton et al.	quenched Lattice	NLO	$\bar{m}_s(2GeV) = 128(18)$	$\Lambda^5 = 240. \pm 90.$

Modified Minimal Subtraction scheme or its corresponding Yukawa coupling related to it through the vev of the Higgs, $\bar{m}(\mu) = v(\mu)\bar{h}(\mu)$.

For the light quarks, up, down and strange, chiral perturbation theory^{3,4} provide us a powerful tool for determining renormalization group invariant quark mass ratios. The absolute values, usually the running mass at $1(GeV)$, can be extracted from different QCD Sum Rules^{5,6,7,8,9} or for the strange quark mass from lattice¹⁰. For the heavy quarks, bottom and charm, we can deal either with QCD Sum Rules^{11,12,13,14}, lattice calculations^{15,16,17,18} or may be soon for the bottom quark with jet physics at LEP^{20,21}. For the top quark we have the recent measurements from CDF and DØ at FERMILAB²⁴ I will identify it with the pole mass.

I have summarized in tables 1 and 2 all of these recent determinations of the quark masses. Of course the final result depends on the strong gauge coupling constant used in the analysis, for this reason I quote it too. In the running I will take $\alpha_s^{(5)}(M_Z) = 0.118 \pm 0.006$ for masses obtained from QCD Sum Rules but

Table 2: Recent determinations of the heavy quark masses from QCD Sum Rules, lattice and FERMILAB.

S. Narison	QSSR Ψ, Υ	NLO	$\bar{m}_b(M_b) = 4.23(4)$ $\bar{m}_c(M_c) = 1.23^{+(4)}_{-(5)}$ $M_b = 4.62(2)$ $M_c = 1.42(3)$	$\alpha_s(M_Z) = 0.118(6)$
S. Narison	non-rel Lapl.c.SR	NLO	$M_b^{NR} = 4.69^{+(3)}_{-(2)}$ $M_c^{NR} = 1.45^{+(5)}_{-(4)}$	$\alpha_s(M_Z) = 0.118(6)$
Dominguez et al.	rel,non-rel Lapl.c.SR $J/\Psi, \Upsilon$	LO $1/m_q^2$	$M_b = 4.70(7)$ $M_c = 1.46(7)$	$\Lambda^4 = 200 - 300$ $\Lambda^5 = 100 - 200$
Titard Ynduráin	$q\bar{q}$ potential		$\bar{m}_b(\bar{m}_b) = 4.397^{+(18)}_{-(33)}$ $\bar{m}_c(\bar{m}_c) = 1.306^{+(22)}_{-(35)}$	$\alpha_s(M_Z) = 0.117(5)$
M. Neubert	QCD SR	NLO $1/m_q$	$M_b = 4.71(7)$ $M_c = 1.30(12)$	

Crisafulli et al. ¹⁵	Lattice in B-meson	$\bar{m}_b(\bar{m}_b) = 4.17(6)$	
Crisafulli et al. ¹⁶	Lattice in B-meson	$\bar{m}_b(\bar{m}_b) = 4.15(7)$	
Davies et al.	NRQCD + leading rel and Lattice spacing, $b\bar{b}$	$M_b = 5.0(2)$ $\bar{m}_b(M_b) = 4.0(1)$	$\alpha_{\overline{MS}}^{(5)} = 0.115(2)$
El-Khadra Mertens	Fermilab action in quenched Lat	$M_c = 1.5(2)$	

CDF	$M_t = 176. \pm 8.(stat) \pm 10.(sys)$	<i>mean</i>
DØ	$M_t = 199.^{+19.}_{-21.}(stat) \pm 22.(sys)$	$M_t = 180. \pm 12.$

for lattice masses I will run with the lattice¹⁷ result $\alpha_s^{(5)}(M_Z) = 0.115 \pm 0.002$. This values are consistent with almost all the references. For those that differ an update is needed but this is beyond the goals of this paper. For instance, S. Narison¹¹ makes two different determinations for the bottom and the charm quark masses. In the first, and for the first time, he gets directly the running mass avoiding then the renormalon ambiguities associated with the pole mass. The second one, from non-relativistic Laplace Sum Rules, is in fact an update of the work of Dominguez et al.¹².

The $O(\alpha_s^2)$ strong correction to the relation between the perturbative pole mass and the running mass was calculated in¹

$$\frac{M}{\bar{m}(M)} = 1 + \frac{4}{3} \frac{\alpha_s(M)}{\pi} + K \left(\frac{\alpha_s(M)}{\pi} \right)^2 + O(\alpha_s^3(M)), \quad (1)$$

where $K_t \simeq 10.95$ for the top quark, $K_b \simeq 12.4$ for the bottom and $K_c \simeq 13.3$ for the charm. As pointed out by S. Narison¹¹ equation 1 is consistent with three loops running but for two loop running we can drop the $O(\alpha_s^2)$ term. Recently, the electroweak correction to the relation between the perturbative pole mass and the Yukawa coupling has been calculated². However, this correction is small, for instance for the top quark it is less than 0.5% in the *SM* for a mass of the Higgs lower than $600(GeV)$ and at most 3.% for $M_H \simeq 1(TeV)$, and for consistency one has to include it only if two loop electroweak running is done.

Instead of expressing the solution of the QCD renormalization group equations for the strong gauge coupling constant and the quark masses in terms of Λ_{QCD} we can solve the running as an expansion in the strong coupling constant at one loop¹⁹. At three loops

$$\alpha_s(\mu) = \alpha_s^{(1)}(\mu) \left(1 + c_1(\mu) \alpha_s^{(1)}(\mu) + c_2(\mu) (\alpha_s^{(1)}(\mu))^2 \right), \quad (2)$$

$$\bar{m}(\mu) = \bar{m}^{(1)}(\mu) \left(1 + d_1(\mu) \alpha_s^{(1)}(\mu) + d_2(\mu) (\alpha_s^{(1)}(\mu))^2 \right), \quad (3)$$

where $\alpha_s^{(1)}(\mu)$ and $\bar{m}^{(1)}(\mu)$ are the one loop solutions

$$\alpha_s^{(1)}(\mu) = \frac{\alpha_s(\mu_0)}{1 + \alpha_s(\mu_0) \beta_0 t}, \quad \bar{m}^{(1)}(\mu) = \bar{m}(\mu_0) K(\mu)^{-2\gamma_0/\beta_0}, \quad (4)$$

with $t = 1/(4\pi) \log \mu^2/\mu_0^2$, $K(\mu)$ the ratio $K(\mu) = \alpha_s(\mu_0)/\alpha_s^{(1)}(\mu)$, and

$$c_1(\mu) = -b_1 \log K(\mu),$$

$$\begin{aligned}
c_2(\mu) &= b_1^2 \log K(\mu) [\log K(\mu) - 1] - (b_1^2 - b_2) [1 - K(\mu)], \\
d_1(\mu) &= -\frac{2\gamma_0}{\beta_0} [(b_1 - g_1) [1 - K(\mu)] + b_1 \log K(\mu)], \\
d_2(\mu) &= \frac{\gamma_0}{\beta_0^2} \left\{ [\beta_0(b_2 - b_1^2) + 2\gamma_0(b_1 - g_1)^2] [1 - K(\mu)]^2 \right. \\
&+ \beta_0(g_2 - b_1 g_1) [1 - K^2(\mu)] \\
&+ \left. \left[4\gamma_0 b_1(b_1 - g_1) [1 - K(\mu)] - 2\beta_0 b_1 g_1 + b_1^2(\beta_0 + 2\gamma_0) \log K(\mu) \right] \log K(\mu) \right\},
\end{aligned} \tag{5}$$

where

$$b_1 = \frac{\beta_1}{4\pi\beta_0}, \quad b_2 = \frac{\beta_2}{(4\pi)^2\beta_0}, \quad g_1 = \frac{\gamma_1}{\pi\gamma_0}, \quad g_2 = \frac{\gamma_2}{\pi^2\gamma_0}, \tag{6}$$

are the ratios of the well known beta and gamma functions in the \overline{MS} scheme

$$\begin{aligned}
\beta_0 &= 11 - \frac{2}{3}N_F, \quad \gamma_0 = 2, \\
\beta_1 &= 102 - \frac{38}{3}N_F, \quad \beta_2 = \frac{1}{2} \left(2857 - \frac{5033}{9}N_F + \frac{325}{27}N_F^2 \right), \\
\gamma_1 &= \frac{101}{12} - \frac{5}{18}N_F, \quad \gamma_2 = \frac{1249}{32} - \frac{277 + 180\zeta(3)}{108}N_F - \frac{35}{648}N_F^2,
\end{aligned} \tag{7}$$

and $\zeta(3) = 1.2020569\dots$ is the Riemann zeta-function. Our initial condition for the strong coupling constant will be $\alpha_s(M_Z)$. Then we will run α_s from M_Z to lower scales, i.e. for instance $\mu_0 = M_Z$ or the upper threshold. On the other side, for the masses we will run from low to higher scales then we need the inverted version of equation 3

$$\bar{m}(\mu_0) = \bar{m}(\mu) K(\mu)^{2\gamma_0/\beta_0} \left(1 - d_1(\mu) \alpha_s^{(1)}(\mu) + (d_1^2(\mu) - d_2(\mu)) (\alpha_s^{(1)}(\mu))^2 \right). \tag{8}$$

The beta and gamma functions depend on the number of flavours N_F therefore we have to decide where we have five where we have four flavours. The trick is as was done by ²² and recently corrected by ²³ to built below the heavy quark threshold an effective theory where the heavy quark has been integrated out. Imposing agreement of both theories, the full and the effective, at low energies they wrote μ dependent matching conditions that express the

parameters of the effective theory, with $N - 1$ quark flavours, as a perturbative expansion in terms of the parameters of the full theory with N flavours

$$\begin{aligned}\alpha_s^{N-1}(\mu) &= \alpha_s^N(\mu) \left[1 + \frac{x}{6} \frac{\alpha_s^N(\mu)}{\pi} + \frac{1}{12} \left(\frac{x^2}{3} + \frac{11x}{2} + \frac{11}{6} \right) \left(\frac{\alpha_s^N(\mu)}{\pi} \right)^2 \right], \\ \bar{m}_l^N(\mu) &= \bar{m}_l^{N-1}(\mu) \left[1 - \frac{1}{12} \left(x^2 + \frac{5x}{3} + \frac{89}{36} \right) \left(\frac{\alpha_s^{N-1}(\mu)}{\pi} \right)^2 \right],\end{aligned}\quad (9)$$

with $x = \log \bar{m}^2(\mu)/\mu^2$, where $\bar{m}(\mu)$ is the mass of the heavy quark we decouple at the energy scale μ and $\bar{m}_l(\mu)$ are the masses of the light quarks. This matching conditions make the strong coupling constant and the mass of the light quarks discontinuous at the thresholds. However, taking the matching in this way we ensure, as pointed out explicitly in ¹⁹, that the final result is independent of the particular matching point we choose for passing the threshold. As it is independent, the easiest way to implement a heavy quark decoupling is to take the threshold as the running mass at the running mass scale, i.e. $\mu_{th} = \bar{m}(\bar{m})$ or equivalently $x = 0$, then the discontinuity appears only at two loops matching.

Table 3: Running at the NLO and NNLO of the top quark mass to M_Z , $\alpha_s^{(5)}(M_Z) = 0.118 \pm 0.006$, $\alpha_s^{(6)}(M_Z) = 0.117 \pm 0.006$.

	$\bar{m}_t(M_t)$	$\bar{m}_t(\bar{m}_t)$	$\bar{m}_t(M_Z)$
NLO	$172. \pm 12.$	$173. \pm 12.$	$182. \pm 13.$
NNLO	$170. \pm 12.$	$171. \pm 12.$	$180. \pm 13.$

I have summarized in tables 3, 4 and 5 the result for the running of the quark masses to M_Z . I mean by NLO connection between the perturbative pole mass and the running mass dropping the $O(\alpha_s^2)$ term, running to two loops and matching at one loop, i.e. strong gauge coupling and masses continuous at $\mu_{th} = \bar{m}(\bar{m})$. Three loops running and matching as expressed in equation 9 with $x = 0$ correspond to NNLO. For consistency with the original works we can only do the running for the bottom and the charm quarks just to NLO. For the light quarks the running is consistent to NNLO using the threshold masses, $\bar{m}(\bar{m})$, of the bottom and charm quarks determined at NLO. I propagate the errors in the running in such a way we maximize them.

It is informative to notice that the running mass of the top quark is shifted about $7(GeV)$ down from its perturbative pole mass that is of the order of its

Table 4: Running at the NLO of the bottom and charm quarks masses to M_Z and running masses at the running mass scale needed for thresholds. For masses extracted from QCD SR $\alpha_s^{(5)}(M_Z) = 0.118 \pm 0.006$, for lattice $\alpha_s^{(5)}(M_Z) = 0.115 \pm 0.002$

	$\bar{m}(\bar{m})$	$\bar{m}(M_Z)$
S. Narison	$\bar{m}_b(\bar{m}_b) = 4.29 \pm 0.04$ $\bar{m}_c(\bar{m}_c) = 1.28 \pm 0.04$	$\bar{m}_b(M_Z) = 2.97 \pm 0.13$ $\bar{m}_c(M_Z) = 0.52 \pm 0.09$
S. Narison	$\bar{m}_b(\bar{m}_b) = 4.35 \pm 0.05$ $\bar{m}_c(\bar{m}_c) = 1.31 \pm 0.06$	$\bar{m}_b(M_Z) = 3.03 \pm 0.13$ $\bar{m}_c(M_Z) = 0.54 \pm 0.10$
S. Titard F.J. Ynduráin	$\bar{m}_b(\bar{m}_b) = 4.397^{+0.018}_{-0.033}$ $\bar{m}_c(\bar{m}_c) = 1.306^{+0.022}_{-0.035}$	$\bar{m}_b(M_Z) = 3.07 \pm 0.11$ $\bar{m}_c(M_Z) = 0.52 \pm 0.08$
M. Neubert	$\bar{m}_b(\bar{m}_b) = 4.37 \pm 0.09$ $\bar{m}_c(\bar{m}_c) = 1.17 \pm 0.12$	$\bar{m}_b(M_Z) = 3.04 \pm 0.17$ $\bar{m}_c(M_Z) = 0.45 \pm 0.14$
Crisafulli et al. ¹⁵	$\bar{m}_b(\bar{m}_b) = 4.17 \pm 0.06$	$\bar{m}_b(M_Z) = 2.93 \pm 0.08$
Crisafulli et al. ¹⁶	$\bar{m}_b(\bar{m}_b) = 4.15 \pm 0.07$	$\bar{m}_b(M_Z) = 2.91 \pm 0.09$
El-Khadra et al.	$\bar{m}_c(\bar{m}_c) = 1.36 \pm 0.19$	$\bar{m}_c(M_Z) = 0.61 \pm 0.15$
Davies et al.	$\bar{m}_b(\bar{m}_b) = 4.13 \pm 0.11$	$\bar{m}_b(M_Z) = 2.89 \pm 0.12$
<i>mean</i>	$\bar{m}_b(\bar{m}_b) = 4.33 \pm 0.06$ $\bar{m}_c(\bar{m}_c) = 1.30 \pm 0.08$	$\bar{m}_b(M_Z) = 3.00 \pm 0.12$ $\bar{m}_c(M_Z) = 0.52 \pm 0.10$

error. Therefore it is important to clarify which mass CDF and $D\bar{D}$ are talking about. I have decoupled the top quark at M_Z otherwise it makes no sense to run the top down. This fact shifts down slightly the strong coupling constant in M_Z , from $\alpha_s^{(5)}(M_Z) = 0.118 \pm 0.006$ we get $\alpha_s^{(6)}(M_Z) = 0.117 \pm 0.006$ but has no effect on the masses because the errors screen the difference between the theory with 5 and 6 flavours. Curiously the running of the top to M_Z cancels the difference between the running and the pole mass.

One has to be very careful in comparing the running of the masses obtained from QCD Sum Rules and those obtained from lattice because I took different values for the strong coupling constant at M_Z . In addition, we have to remember that the error in the running is dominated by the error in the strong coupling constant. However it is impressive to notice the good agreement of the results obtained in lattice^{17,15} with the running of the masses from QCD Sum Rules. Even, the APE-Collaboration¹⁵, has improved recently its result by increasing the statistics on the lattice¹⁶.

We can now play a game combining the light quark masses of table 5 with the ratios obtained from χ PT. The mean value of the strange quark mass together with the Bijmans et al.⁵ result gives

$$\frac{2\bar{m}_s}{\bar{m}_u + \bar{m}_d} = 33. \pm 12., \quad (10)$$

in agreement with the χ PT result. Being conservative we can also get for the up and down quarks $\bar{m}_u(1GeV) = (3. \pm 2.)MeV$ and $\bar{m}_d(1GeV) = (9. \pm 2.)MeV$ that translate into $\bar{m}_u(M_Z) = (1.5 \pm 1.2)MeV$ and $\bar{m}_d(M_Z) = (4.1 \pm 1.7)MeV$.

Table 5: Running of the light quark masses to M_Z . For masses extracted from QCD SR $\alpha_s^{(5)}(M_Z) = 0.118 \pm 0.006$, for lattice $\alpha_s^{(5)}(M_Z) = 0.115 \pm 0.002$. First box is NLO running, second and third boxes are NNLO running.

	$\bar{m}(1GeV)$	$\bar{m}(M_Z)$
S. Narison	$\bar{m}_s(1GeV) = 222. \pm 22.$	$\bar{m}_s(M_Z) = 105. \pm 28.$
Allton et al.	$\bar{m}_s(1GeV) = 156. \pm 17.$	$\bar{m}_s(M_Z) = 78. \pm 15.$

	$\bar{m}(1GeV)$	$\bar{m}(M_Z)$
S. Narison	$\bar{m}_s(1GeV) = 197. \pm 29.$	$\bar{m}_s(M_Z) = 88. \pm 31.$
Jamin / Münz	$\bar{m}_s(1GeV) = 189. \pm 32.$	$\bar{m}_s(M_Z) = 85. \pm 32.$
Chetyrkin et al.	$\bar{m}_s(1GeV) = 171. \pm 15.$	$\bar{m}_s(M_Z) = 75. \pm 23.$
<i>mean</i>	$\bar{m}_s(1GeV) = 186. \pm 30.$	$\bar{m}_s(M_Z) = 83. \pm 30.$

Bijnens et al.	$(\bar{m}_u + \bar{m}_d)(1GeV) = 12.0. \pm 2.5$	$(\bar{m}_u + \bar{m}_d)(M_Z) = 5.4 \pm 2.2$
Ioffe et al.	$(\bar{m}_d - \bar{m}_u)(1GeV) = 3. \pm 1.$	$(\bar{m}_d - \bar{m}_u)(M_Z) = 1.4 \pm 0.7$

To summarize, the running of the quark masses to the M_Z energy scale gives us a running top mass that is around its perturbative pole mass, $\bar{m}_t(M_Z) = (180. \pm 13.)GeV$, for the bottom and the charm quark we get $\bar{m}_b(M_Z) = (3.00 \pm 0.12)GeV$ and $\bar{m}_c(M_Z) = (0.52 \pm 0.10)GeV$ respectively, while for the strange quark we have a result affected by a big error $\bar{m}_s(M_Z) = (83. \pm 30.)MeV$. The same happens for the up and down quarks, we get $\bar{m}_u(M_Z) = (1.5 \pm 1.2)MeV$ and $\bar{m}_d(M_Z) = (4.1 \pm 1.7)MeV$.

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